Technical Comment

Comment on "Turbulent Skin Friction for Tapered Wings"

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THE use of the skin-friction law

$$C_f = 0.455/(\log_{10} R)^{2.58} \quad (1)$$

by Ross[1] and Barkhem[2] makes the evaluation of the integral defining the average skin friction coefficient difficult. The difficulty can be avoided by using any expression for the skin friction at the chord Reynolds number, R_r, but approximating the variation in the range of integration by a power law.

The integral (using Ross' notation) can be written as

$$C_f = C_f(R_r)\frac{2}{1+\lambda}\int_0^1 [1+(1-\lambda)\xi]\frac{C_f(\xi)}{C_f(1)}\,d\xi \quad (2)$$

The distinction between $C_f(R_r)$ outside the integral and $C_f(1)$ inside is crucial. Outside the integral an accurate "absolute" value of C_f evaluated at R_r is needed, but inside only an expression for the ratio of $C_f(\xi)/C_f(1)$ over a limited range of Reynolds number is required.

If $C_f(R)$ is given by a power law

$$C_f = k/R^n \quad (3)$$

then it is easily seen that

$$C_f(\xi)/C_f(1) = [1-(1-\lambda)\xi]^{-n} \quad (4)$$

and the integral can be evaluated analytically to give

$$C_f/C_f(R_r) = [2/(2-n)][(1-\lambda^{2-n})/(1-\lambda^2)] \quad (5)$$

Hopkins[3] shows by l'Hospital's rule that

$$\lim_{\lambda\to 1}(1-\lambda^{2-n})/(1-\lambda^2) = (2-n)/2 \quad (6)$$

so that Eq. (5) reduces to the correct result $C_f = C_f(R_r)$ for a straight wing.

The point of this comment is that Eq. (4) is a legitimate approximation for any friction coefficient-Reynolds number relation. It can be shown[4] that a function $f(x)$ can be approximated in the neighborhood of x_0 by

$$f(x) = ax^e \quad (7)$$

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Table 1 Comparison of $C_f/C_f(R_r)$

λ	$R_r = 10^7$ Simpson's Rule	Analytic	$R_r = 10^9$ Simpson's Rule	Analytic	$n = 1/7$ Analytic
0.1	1.085	1.082	1.064	1.063	1.073
0.3	1.066	1.064	1.050	1.049	1.060
0.5	1.045	1.044	1.035	1.034	1.040
0.7	1.026	1.026	1.020	1.020	1.023
0.9	1.008	1.008	1.006	1.006	1.007

where

$$e = (x/f)(df/dx)|_{x=x_0} \quad (8a)$$

and

$$a = f(x_0)/x_0^n \quad (8b)$$

Applying this to Eq. (1) gives

$$n = 2.58/\ln R_r = 1.205/\log_{10} R_r \quad (9)$$

Table 1 compares the ratio of $C_f/C_f(R_r)$ obtained for Reynolds numbers of 10^7 and 10^9 by using Simpson's rule to evaluate the integral with $C_f(\xi)$ given by Eq. (1) with the values obtained by using Eqs. (5) and (9). The small differences have no practical significance. The last column using Eq. (5) with $n = 1/7$ shows that this is quite adequate for this Reynolds number range.

The use of Eq. (5) allows more accurate evaluations of $C_f(R_r)$ than simple skin friction laws such as Eq. (1) or Eq. (3) to be employed. Collar's expression[5] which takes the laminar contribution into account or simple airfoil methods such as the one given by Thwaites[6] can be applied. Also, it is possible to obtain analytic results for other planforms.

References

[1] Ross, R., "Turbulent Skin Friction for Tapered Wings," *Journal of Aircraft*, Vol. 8, No. 10, Oct. 1971, pp. 836–837.

[2] Barkhem, A., "Skin-Friction Formula for Tapered and Delta Wings," *Journal of Aircraft*, Vol. 6, No. 3, May–June 1969, p. 284.

[3] Hopkins, E. J., "Some Effects of Planform Modification on the Skin Friction Drag" *AIAA Journal*, Vol. 2, No. 2, Feb. 1964, pp. 413–1914.

[4] Squire, W., "Power Laws," *Proceedings of the West Virginia Academy of Science*, Vol. 35, 1963, pp. 196–199.

[5] Collar, A. R., "A Closed Formula for the Drag of a Flat Plate with Transition in the Absence of a Pressure Gradient," *Journal of the Royal Aeronautical Society*, Vol. 64, No. 589, Jan. 1960, pp. 38–39.

[6] Thwaites, B., *Incompressible Aerodynamics*, 1st ed., Clarendon Press, Oxford, 1960, pp. 179–184.